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Higher-Order Risk Preferences

Consequences for Test and Treatment Thresholds and Optimal Cutoffs

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Stefan Felder and Thomas Mayrhofer[1]

Higher-Order Risk Preferences – Consequences for Test and Treatment Thresholds and Optimal Cutoffs

Abstract

Higher-order risk attitudes include risk aversion, prudence, and temperance. This paper analyzes the effects of such preferences on medical test and treatment decisions, represented either by test and treatment thresholds or – if the test characteristics are endogenous – by the optimal cutoff value for testing. For a risk-averse decision maker, treatment is a risk reducing strategy since it prevents the low health outcome that forgoing treatment yields in the sick state. As compared to risk neutrality, risk aversion thus reduces both the test and the treatment threshold and decreases the optimal cutoff. Prudence is relevant if a comorbidity risk applies in the sick state. It leads to even lower thresholds and a lower optimal cutoff. Finally, temperance plays a role if the comorbidity risk is left-skewed. It lowers the thresholds and the optimal cutoff even further. These findings suggest that diagnostics in low prevalence settings (e.g. screening) are considered more beneficial when higher-order risk preferences are taken into account.

JEL Classification: D81, I10

Keywords: Medical decision making; diagnostic risk; test and treatment thresholds; optimal cutoff; risk aversion; prudence

October 2011

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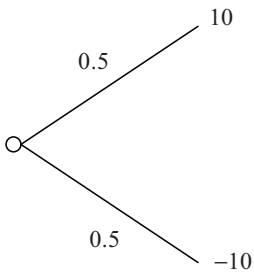
1 Introduction

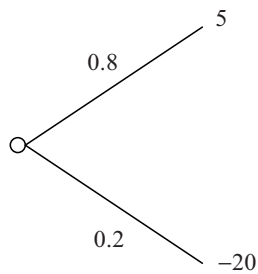
In 1975, Stephen G. Pauker and Jerome P. Kassirer introduced the concept of diagnostic risk and its impact on medical treatment decisions. They showed how – depending on the a priori probability of illness – a decision maker should decide between treatment and no treatment in the absence of a diagnostic test. Five years later, the same authors developed a threshold analysis (Pauker and Kassirer 1980), extending their earlier work by introducing the possibility of diagnostic testing. They proposed a method for valuing a diagnostic test and a rule, also dependent on the a priori probability of illness, for when to use the test.

In Pauker and Kassirer's original analysis, outcomes from treatments take the form of survival and mortality rates, i.e. increased survival for sick patients and reduced survival for healthy patients. This approach is equivalent to an expected utility framework where the decision maker is risk neutral. In this paper, we bring the von Neumann-Morgenstern hypothesis to its full use and consider higher-order risk preferences, i.e. risk aversion, prudence, and temperance, which concern the second, third, and fourth derivatives of the decision maker's utility function. We analyze how higher-order risk preferences affect the thresholds in the a priori probability of illness that define the prevalence range in which testing is indicated. Furthermore, we demonstrate how higher-order risk preferences influence the choice of the positivity criterion, i.e. the optimal cutoff value of a diagnostic test when test characteristics are endogenous.

We develop a model which considers a left-skewed comorbidity risk attached to a patient's sick state. This risk is assumed to be exogenous, i.e. it occurs irrespective of whether the physician treats the patient or not. Fig. 1 illustrates the difference between a symmetric and a left-skewed comorbidity risk. The outcomes represent gains or losses in the sick patient's health level. The two prospects each have an expected mean of zero and the same variance. The distribution of the left-hand prospect is symmetric while the right-hand one is left-skewed. If the comorbidity risk is part of a larger prospect – the health outcomes of which also depend on the treatment decision – then symmetry in the comorbidity risk will influence the overall skewness while left-skewness will also impact the overall kurtosis. Since, loosely speaking, prudence refers to skewness and temperance to kurtosis, we will show that a temperate decision maker reacts most strongly to a left-skewed comorbidity risk, followed by the prudent and then the risk-averse decision maker. These results are relevant to medical decision making, since left-skewness of risks is common in medicine.



Symmetric comorbidity risk



Left-skewed comorbidity risk

Fig. 1: Symmetric and left-skewed comorbidity risks

The remainder of the paper is organized as follows. We start Section 2 with a base model which captures risk neutrality and risk aversion. We then add a comorbidity risk to the model, which allows us to address prudence and temperance. The different higher-order risk preferences can be compared with a measure which we develop in Section 2. The measure is the ratio of the utility gain from treatment in the sick state and the utility loss from treatment in the healthy state, representing the trade-off a physician faces when deciding for or against treatment. Section 3 analyzes how the treatment decision changes when moving from risk neutrality to higher-order risk preferences. Section 4 adds diagnostic testing and presents comparative statics for the effects of higher-order risk preferences on the test and treatment thresholds. Section 5 discusses the analysis for endogenous test outcomes. Section 6 presents a clinical example and section 7 discusses our findings and concludes.

2 Risk Aversion, Prudence, and Temperance

Our focus here is on the diagnostic risk inherent in the decision on medical treatment. In our analyses we do not differentiate between physicians and patients, but rather assume that decision makers (DMs) decide purely in the interest of their patients.

The patient's health state i is unknown; he can be sick or healthy, $i = s, h$, where p describes the probability of the sick state. The DM must decide whether or not to administer treatment. He can also use diagnostics and make the treatment decision dependent on the test outcome j, $j = +, -$. A negative test result ($j = -$), is associated with the extreme health outcomes, i.e.

the best outcome in the healthy state ($i = h$) and the worst outcome in the sick state ($i = s$). The intermediate outcomes are associated with a positive test result ($j = +$). The four possible health outcomes H_i^j ($i = h,s$, $j = +,-$) can thus be ranked as follows: $H_h^- > H_h^+, H_s^+ > H_s^-$. The DM's elementary utility function is given by $U\left(H_i^j\right)$, which is assumed to have a positive slope: $U'\left(H_i^j\right) \equiv \partial U\left(H_i^j\right) / \partial H_i^j > 0$. Positive marginal utility implies that for a negative test result, for instance, utility in the healthy state exceeds utility in the sick state: $U\left(H_h^-\right) > U\left(H_s^-\right)$.

A positive test result followed by treatment will be beneficial in the sick state and harmful in the healthy state. We define

(1) $G = U\left(H_s^+\right) - U\left(H_s^-\right) > 0$ and $L = U\left(H_h^+\right) - U\left(H_h^-\right) < 0$

as the utility gain from treatment in the sick state and the utility loss from treatment in the healthy state.

First, we distinguish between a risk-averse DM *A* and a risk neutral DM *N*. The latter is indifferent between a certain outcome and an uncertain outcome of the same expected value, while the former prefers the certain outcome to the uncertain prospect.

The two DMs' different risk attitudes can be illustrated by the curvature of their utility functions. *N* has a linear utility function with a second derivative equal to zero: $U_N''\left(H_i^j\right) \equiv \partial^2 U\left(H_i^j\right) / \left(\partial H_i^j\right)^2 = 0$. *A* has a concave utility function with a negative second derivative, i.e. $U_A''\left(H_i^j\right) < 0$. *A* values a marginal improvement in health more at low levels than at high levels, while *N* is indifferent between marginal health improvements at different health levels. *A* can also be seen as more inclined to prevent the worst outcome than *N*.

Under the expected utility hypothesis, decisions are independent of positive linear transformations of the utility function. Without loss of generality, we can therefore assume that *N* and *A* value the extreme health states H_s^- and H_h^- equally (see Fig. 2). Due to the concavity of *A*'s utility function, his utility gain from treatment in the sick state *G* is higher, while his utility loss in absolute terms from treatment in the healthy state *L* is lower than *N*'s.

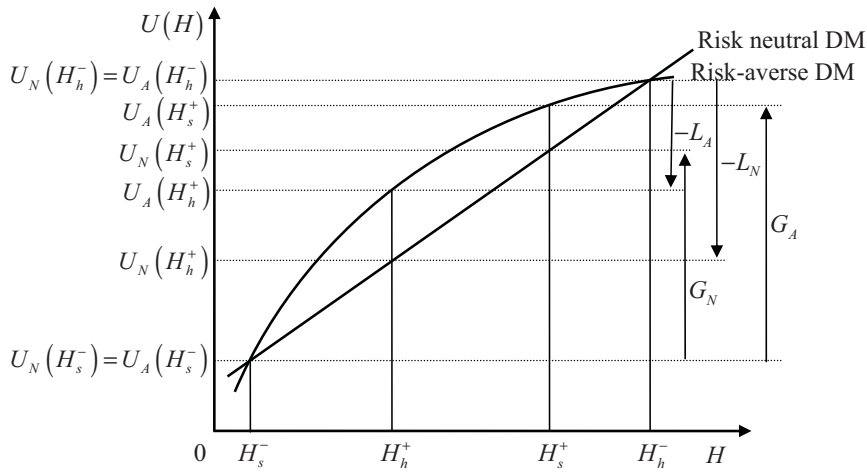


Fig. 2: The health utility functions for risk neutral and risk-averse decision makers

From these considerations, we can derive $G_A > G_N$, $-L_A < -L_N$, or, given (1), $-G_A/L_A > -G_N/L_N$.

In order to address prudence and temperance we introduce a background risk $\tilde{m}$, which can stand for comorbidity or the severity of an illness. As a consequence, health in the sick state H_s becomes a random variable: $H_s + \tilde{m}$. We assume that $E[\tilde{m}] = 0$. Furthermore, the background risk is exogenous – its outcome is independent of the DM's treatment decision.

Given the background risk, utilities for the sick state need to be written as expected values: $EU(H_s^- + \tilde{m})$ and $EU(H_s^+ + \tilde{m})$. As the comorbidity risk refers to the sick state only, utilities in the healthy state remain the same, $U(H_h^-)$ and $U(H_h^+)$. The same goes for *L*, the utility loss from treatment in the healthy state. We will now show that the expected utility gain from treatment *G* increases in the presence of a comorbidity risk.

Provided that $\tilde{m}$ is sufficiently small, we can apply a Taylor expansion to approximate $EU(H_s + \tilde{m})$:

$$EU(H_s+\tilde{m}) \cong U(H_s)+U'(H_s)\cdot \underbrace{E[\tilde{m}]}_{0}+\frac{1}{2}U''(H_s)\cdot \underbrace{E\left[(\tilde{m}-E[\tilde{m}])^2\right]}_{\sigma^2_{\tilde{m}}}$$

$$\cong U(H_s)+\frac{\sigma^2_{\tilde{m}}}{2}U''(H_s),$$

where $\sigma^2_{\tilde{m}}$ is the variance of $\tilde{m}$. The difference in the expected utilities of 'treatment' and 'no treatment' then becomes

$$\begin{aligned} EG &\cong \underbrace{U\left(H_s^+\right)-U\left(H_s^-\right)}_{G}+\frac{\sigma^2_{\tilde{m}}}{2}\left(U''\left(H_s^+\right)-U''\left(H_s^-\right)\right) \\ &\cong G+\frac{\sigma^2_{\tilde{m}}}{2}\left(U''\left(H_s^+\right)-U''\left(H_s^-\right)\right). \end{aligned} \tag{2}$$

As $\sigma^2_{\tilde{m}}$ is positive, the sign of the difference $EG-G$ depends on difference in the second derivative of the utility function, $U''\left(H_s^+\right)-U''\left(H_s^-\right)$, which in turn is determined by the third derivative of the utility function, $U'''\left(H_i^j\right)$.

The third derivative of the utility function is obviously zero for a risk neutral DM. Hence, *N* would not react to a background risk with an expected value of zero: $EG_N = G_N$. The same holds for a risk-averse DM whose utility function has a third derivate equal to zero. Here, $EG_A = G_A$.

If a DM's utility function has a positive third derivative, he is called prudent (Kimball 1990). Prudence has a stronger effect on the treatment decision than risk aversion. For a prudent DM *P*, we have $EG_P > G_A$. In other words, prudence increases the utility gain from treatment in the sick state even more than risk aversion. *P* will put more weight on avoiding the extreme negative outcome possible in the situation without treatment.

The concept of temperance was introduced by Kimball (1992) and is illustrated by Eeckhoudt and Schlesinger (2006) as follows. Suppose a DM faces two statistically independent risks. In addition to the comorbidity risk in the sick state described above, he must accept a second zero-mean random variable in either one of the two health states. An individual is called temperate if he prefers the second risk to be attached to the healthy state rather than to the sick state. In other words, he prefers a separation of the two exogenous risks.

Temperance also matters if the comorbidity risk attached to the sick state is left-skewed rather than symmetrically distributed. Holding the expected value of the risk at zero, a left skewed distribution implies that the bad outcome occurs with a low probability, but outweighs the good outcome in magnitude.

With a skewed background risk we have to extend the Taylor expansion up to the third order:

$$\begin{aligned} EU(H_s+\tilde{m}) &\cong U(H_s)+U'(H_s)\cdot \underbrace{E[\tilde{m}]}_{0} \\ &\quad +\frac{U''(H_s)}{2}\cdot \underbrace{E\left[\left(\tilde{m}-E[\tilde{m}]\right)^2\right]}_{\sigma_{\tilde{m}}^2}+\frac{U'''(H_s)}{6}\cdot \underbrace{E\left[\left(\tilde{m}-E[\tilde{m}]\right)^3\right]}_{\gamma_{\tilde{m}}\cdot\left(\sigma_{\tilde{m}}^2\right)^{3/2}=\gamma_{\tilde{m}}\cdot\sigma_{\tilde{m}}^3} \\ &\cong U(H_s)+\frac{\sigma_{\tilde{m}}^2}{2}U''(H_s)+\frac{\gamma_{\tilde{m}}\cdot\sigma_{\tilde{m}}^3}{6}U'''(H_s), \end{aligned}$$

where $\gamma_{\tilde{m}}$ denotes the third standardized moment, i.e. skewness, and $\sigma_{\tilde{m}}^3$ the third power of the standard deviation of the background risk. For a temperate DM the (expected) utility gain from treatment in the sick state can then be written as

$$\begin{aligned} EG_T &\cong G+\frac{\sigma_{\tilde{m}}^2}{2}\left(U''\left(H_s^+\right)-U''\left(H_s^-\right)\right)+\frac{\gamma_{\tilde{m}}\cdot\sigma_{\tilde{m}}^3}{6}\left(U'''\left(H_s^+\right)-U'''\left(H_s^-\right)\right) \\ &\cong EG_P+\frac{\gamma_{\tilde{m}}\cdot\sigma_{\tilde{m}}^3}{6}\left(U'''\left(H_s^+\right)-U'''\left(H_s^-\right)\right). \end{aligned} \tag{3}$$

Given that $\gamma_{\tilde{m}}<0$ and $\sigma_{\tilde{m}}^3>0$, temperance compounds the effect of prudence on the expected utility gain from treatment in the sick state, provided that the difference in the third derivative of the utility function is negative, $U'''\left(H_s^+\right)-U'''\left(H_s^-\right)<0$.

The fourth derivative of a temperate individual's utility function is negative, $U^{iv}\left(H_i^j\right)<0$ (see for instance Eeckhoudt et al. 1996). Assuming that the individual is also prudent, so $U'''\left(H_i^j\right)>0$, the difference in the third derivative will be negative, $U'''\left(H_s^+\right)-U'''\left(H_s^-\right)<0$. A left-skewed background risk, $\gamma_{\tilde{m}}<0$, will then lead to an additional increase in the expected utility gain from treatment for the sick, so that $EG_T>EG_P$.

Left-skewness of the background risk would not matter for a DM who is prudent but not temperate. His decision can be explained by the first two moments of the background risk distribution alone.

We can summarize these considerations in the following illustrative way:

(4) $$-EG_T/L_T > -EG_P/L_P > -G_A/L_A > -G_N/L_N,$$

where $L_T = L_P = L_A$. When considering the utility gain from treatment in the sick state and the utility loss from treatment in the healthy state, the risk neutral DM is indifferent to marginal changes in the health levels in these two states. The risk-averse DM values a marginal increase in health higher in the sick state than in the healthy state. When facing a background risk, the prudent DM puts even more weight on improving health in the sick state, and the maximal weight is attached to the sick state by the temperate DM (when the background risk is left-skewed).

3 Treatment Decision without a Diagnostic Test: The Critical Prevalence Rate

In order to estimate the value of information provided by a diagnostic test, one must first consider the DM's decision whether or not to treat in a situation in which no test is available. In this base case without comorbidity risk, expected utility as a function of *j*, with $j = +$ for treatment and $j = -$ for no treatment, becomes

(5) $$EU^j(p) = pU\left(H_s^j\right) + (1-p)U\left(H_h^j\right).$$

There is an a priori probability of illness $\tilde{p}$ at which the DM is indifferent between treatment and no treatment. This threshold, called the critical prevalence rate, satisfies

(6) $$\tilde{p} = \frac{-L}{G-L} = \frac{1}{1-G/L} > 0.$$

For $p < \tilde{p}$, the expected utility of no treatment exceeds the expected utility from treatment, so that not treating is indicated. For $p > \tilde{p}$, by comparison, the optimal strategy is to treat.

This threshold will depend on the DM's risk preferences. We classified the different higher-order risk preferences with respect to the ratio of utility gains and losses from treatment in the sick and healthy states. To find the effects of risk aversion, prudence, and temperance on the critical prevalence rate, we therefore calculate the derivative of (6) with respect to $(G/(-L))$:

(7) $$\frac{\partial \tilde{p}}{\partial(-G/L)} = -\tilde{p}^2 < 0.$$

In other words, if either the utility gain from treatment in the sick state increases or the loss from treatment in the healthy state decreases (in absolute terms), the critical prevalence rate decreases and treatment is indicated at lower a priori probabilities. This makes perfect sense: if either the potential benefit from treatment increases or the potential harm decreases, the treatment option becomes more favorable.

Now, the utility gain from treatment increases for a risk-averse DM compared to a risk neutral DM, $G_A > G_N$, while the utility loss from treatment decreases, $-L_A < -L_N$. This leads to $-G_A/L_A > -G_N/L_N$, and thus to a lower critical prevalence rate for *A* than for *N*, $\tilde{p}_A < \tilde{p}_N$. *A* uses the treatment strategy as an insurance device (it reduces the spread between the possible health states), and thus opts for treatment at lower prevalence rates than *N*.

In the situation with comorbidity risk the third derivative of the utility function appears to be relevant as well. Since $EG_P > G_A$ while $-L_P = -L_A$, it follows that the critical prevalence rate is reduced even further: $\tilde{p}_P < \tilde{p}_A$. Although the background risk is exogenous, the prudent DM braces himself in the face of it and treats earlier than the (merely) risk-averse DM. If the comorbidity risk is left-skewed and the DM is also temperate, we have $EG_T > EG_P$ and thus $\tilde{p}_T < \tilde{p}_P$.

Summing up, we conclude that

(8) $\tilde{p}_T < \tilde{p}_P < \tilde{p}_A < \tilde{p}_N$.

The critical prevalence rate will be lower for a risk-averse DM than for a risk neutral one. Facing comorbidity risk leads to a further reduction of the threshold for prudent DM. Left-skewness of the background risk and temperance reduces the threshold even further.

4 Treatment Decision with a Diagnostic Test: Test and Treatment Thresholds

The availability of a diagnostic test increases the DM's options. Instead of two actions (treating and not treating), he now has the additional option of using diagnostics. The performance of a test depends on its sensitivity *Se* and specificity *Sp*. If the test is employed, we assume that a positive test outcome will lead to treatment while a negative outcome will not. The decision situation for the DM in the base model is shown in Fig. 3.

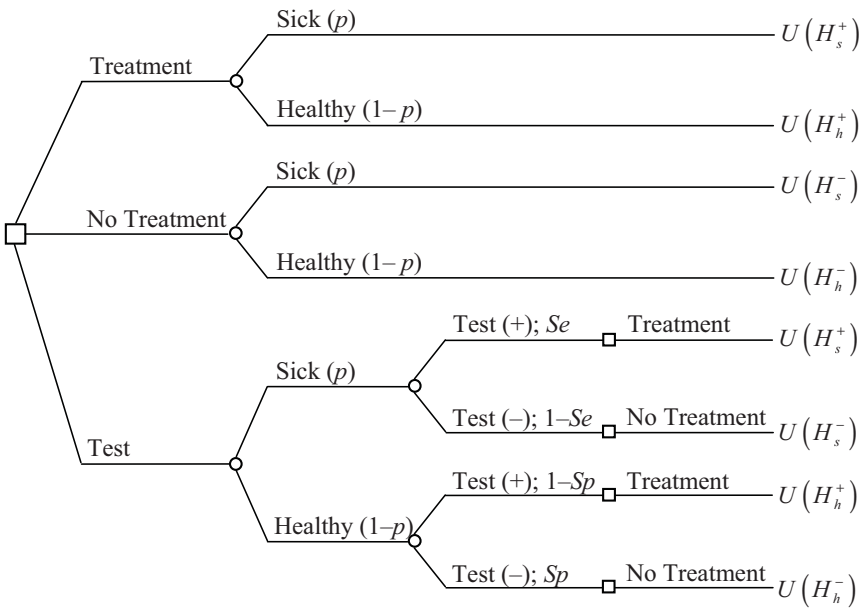


Fig. 3: Decision tree with a diagnostic test

In the base case, the expected utility of a diagnostic test – $EU^{Dx}(p)$ – can be written as follows:

$$
\begin{aligned}
EU^{Dx}(p) &= p\left[Se\cdot U\left(H_s^+\right)+(1-Se)U\left(H_s^-\right)\right]+(1-p)\left[Sp\cdot U\left(H_h^-\right)+(1-Sp)U\left(H_h^+\right)\right] \\
&= p\left[U\left(H_s^-\right)+Se\cdot G\right]+(1-p)\left[U\left(H_h^+\right)-Sp\cdot L\right].
\end{aligned} \tag{9}
$$

The value of diagnostic information is defined as the additional expected utility resulting from the use of the test. The reference situation is the expected utility of not treating for $p<\tilde{p}$, and the expected utility from treatment for $p\geq\tilde{p}$. Given equations (1), (5), and (9), we find for the value of information of a diagnostic test

$$
VI(p)=\begin{cases} EU^{Dx}(p)-EU^{-}(p)=p\cdot Se\cdot G+(1-p)(1-Sp)L & \text{for} \quad 0\leq p<\tilde{p}, \\ EU^{Dx}(p)-EU^{+}(p)=-pG(1-Se)-(1-p)Sp\cdot L & \text{for} \quad \tilde{p}\leq p\leq 1. \end{cases} \tag{10}
$$

Setting the equations of (10) to zero, we obtain the test and treatment thresholds introduced by Pauker and Kassirer (1980):

(11) $\tilde{p}^{Dx} = \dfrac{-(1-Sp)L}{Se \cdot G - (1-Sp)L} = \dfrac{1}{1-LR^{+}\,G/L}$ and $\tilde{p}^{Rx} = \dfrac{-Sp \cdot L}{(1-Se)G - Sp \cdot L} = \dfrac{1}{1-LR^{-}\,G/L}$.

Like the critical prevalence rate, they mark probability thresholds at which the DM is indifferent between two actions. At the test threshold, $\tilde{p}^{Dx}$, he is indifferent between not treating and testing (followed by the treatment decision depending on the test outcome). At the treatment threshold, $\tilde{p}^{Rx}$, he is indifferent between testing and treating without prior testing.

$LR^{+} = Se/(1-Sp)$ denotes the positive and $LR^{-} = (1-Se)/Sp$ the negative likelihood ratio. Since $LR^{+} > 1 > LR^{-} > 0$, $\tilde{p}^{Dx}$ is located below the critical prevalence rate and $\tilde{p}^{Rx}$ above of it: $\tilde{p}^{Dx} < \tilde{p} < \tilde{p}^{Rx}$.

We can differentiate the two new thresholds with respect to $(G/(-L))$ in order to qualify the effects that higher-order risk preferences have on them:

(12) $\dfrac{\partial \tilde{p}^{Dx}}{\partial(-G/L)} = -LR^{+}\left(\tilde{p}^{Dx}\right)^{2} < 0$ and $\dfrac{\partial \tilde{p}^{Rx}}{\partial(-G/L)} = -LR^{-}\left(\tilde{p}^{Rx}\right)^{2} < 0$.

From (4) we can immediately derive the following rank order:

(13) $\tilde{p}_T^{Dx} < \tilde{p}_P^{Dx} < \tilde{p}_A^{Dx} < \tilde{p}_N^{Dx}$ and $\tilde{p}_T^{Rx} < \tilde{p}_P^{Rx} < \tilde{p}_A^{Rx} < \tilde{p}_N^{Rx}$.

The downward shift of the thresholds as we move from a risk neutral DM to a temperate DM can be explained as follows. Treatment is a risk reducing strategy. A risk-averse DM values a test more at low a priori probabilities where the reference strategy in the situation without a test is no treatment. Consequently, *A* will test earlier than *N*, implying a lower test threshold. At high a priori probabilities, the reference strategy is treatment. *A* will sooner not run the risk of a false negative test result and thus treat earlier.

If there is a comorbidity risk, more weight is given to the utility gain in the sick state. This lowers the thresholds even further. A prudent DM will test and treat earlier - and therefore also more often, as the prevalence range where treatment is indicated expands – when faced with a comorbidity risk. Since the comorbidity risk is exogenous, the DM cannot address it directly through his behavior. He will instead reduce his overall risk by choosing treatment, the strategy which reduces his endogenous risk.

Finally, the thresholds are lowest for a temperate DM *T* facing a left-skewed comorbidity risk. For *T* a left-skewed zero-mean background risk implies greater harm than a symmetric one, so that he acts even more prudently by further reducing his endogenous risk.

Note that our results are clear-cut regarding the a priori probability range for treatment $\left(\tilde{p}^{Rx},1\right)$, but not for the a priori probability range for testing $\left(\tilde{p}^{Dx},\tilde{p}^{Rx}\right)$. Since the treatment threshold decreases, the interval $\left(\tilde{p}^{Rx},1\right)$ increases with higher-order risk aversion. This need not be the case for $\left(\tilde{p}^{Dx},\tilde{p}^{Rx}\right)$, because the test threshold decreases too. Thus, whether or not the a priori probability range for testing increases with higher-order risk preferences depends on the relative shift of the two thresholds.

5 Higher-order Risk Preferences and the Optimal Cutoff

Our analysis of the effect of higher-order risk preferences on test and treatment behavior can be extended to include the situation where the DM has to determine the test result by setting a cutoff value. A good example is the prostate specific antigen (PSA) test for the detection of prostate cancer in men. The analysis of a blood sample results in a PSA value which the physician declares as positive or negative depending on his chosen cutoff value. Correspondingly, the test characteristics sensitivity and specificity are no longer given, but determined by the chosen cutoff. The test technology can then be represented by the receiver operating characteristics (ROC) curve, which represents all the feasible maximal sensitivity-specificity pairs. In the simplest case where there is only one marker, such as the PSA value, each possible cutoff value corresponds to one point on the ROC curve. A higher cutoff value increases specificity at the expense of sensitivity. The DM's task is to choose the optimal cutoff value at which the patient's expected utility is maximized. The optimal point along the ROC curve satisfies the following condition[1,2]:

$$(14) \qquad \frac{dSe}{d\left(1-Sp\right)}=-\frac{1}{\Omega G/L},$$

[1] McNeil et al. (1975) and Metz (1978) present a characterization of the optimal point along the ROC curve. Felder et al. (2003) introduce iso value curves which lead to the derivation of the optimal point. Finally, Felder and Mayrhofer (2011, chapter 8) explicitly solve the optimization problem.

[2] Note that with optimally set cutoffs, the test and treatment thresholds disappear and the testing range covers the entire (0,1) prevalence range (see Felder and Mayrhofer 2011, chapter 8).

where Ω is the a priori odds of the illness, $\Omega = p/(1-p)$. Equation (14) shows that the optimal point depends on the prevalence (through the a priori odds) as well as the utility gains and losses from treatment. With an increase in the prevalence rate *p* or in the utility gain *G*, an upward move along the ROC curve is optimal, implying an increase in sensitivity at the expense of specificity. Provided that the marker is positively correlated with the presence of the illness, this implies a lower optimal cutoff. This comparative statics result is intuitive, since the importance of detecting the sick increases in the probability of illness or with a higher utility gain of treatment in the sick state. The opposite applies for an increase in the utility loss *–L,* where a downward move along the ROC curve is optimal, leading to lower sensitivity and increased specificity.

The effect of higher-order risk preferences on the optimal point along the ROC curve can be evaluated as follows:

$$(15) \qquad \frac{\partial\left(\frac{dSe}{d(1-Sp)}\right)}{\partial(-G/L)} = -\frac{1}{\Omega}\left(\frac{1}{G/L}\right)^2 < 0.$$

Denoting the cutoff value with *x* we then derive the following rank order for the optimal (*) cutoffs as a function of higher-order risk preferences:

$$(16) \qquad x_T^* < x_P^* < x_A^* < x_N^*.$$

If the test result shows a lower value than the optimal cutoff, the physician should not treat. By contrast, a higher value should lead to treatment. A lower optimal cutoff then implies earlier treatment. The intuition for the rank order (16) is the same as in the last two sections: Risk aversion puts more weight on the utility gain in the sick state and thus will lead to earlier treatment. Given a background risk, prudence and temperance (for left-skewed background risks) strengthen this effect, causing a further decrease in the optimal cutoff.

6 Clinical Example

For illustrative purposes, we take a clinical example from Sox et al. (2007), chapter nine. A physician examines a 55 year-old male with a headache and progressive unilateral weakness. The physician suspects a brain tumor which can be treated with brain surgery. For simplicity, we consider life expectancy to be the only outcome parameter. If he is healthy the patient can

expect to live another 21 years. Unnecessary brain surgery will reduce his remaining life expectancy to 20 years. If he has a brain tumor and remains untreated he can expect to live for 2 more years only, while brain surgery in this case will increase his remaining life expectancy to 11 years.

First, we calculate the critical prevalence rate for DM with different higher-order risk preferences. Since the utility function of a risk-neutral DM is linear, the utility gain *G* and utility loss *L* from treatment can be expressed in life expectancies: $G_N = 11-2 = 9$ and $L_N = 20-21 = -1$. Therefore, the critical prevalence rate for a risk neutral decision maker will be $\tilde{p}_N = -L_N/(G_N - L_N) = 1/(9+1) = 10.00\%$. Thus, in this situation without further diagnostics, brain surgery is the optimal strategy for a risk neutral DM, provided that the a priori probability of the patient having a brain tumor is at least 10%.

Sox et al. (2007) report the following utilities from life expectancies that were derived using the standard gamble method: $U(21) = 1$; $U(2) = 0.25$; $U(0) = 0$. These utility levels can be approximated by the utility function $U(LE) = LE^{0.6}/6.2$, which represents a so called mixed risk-averse type, i.e. a DM who is risk-averse, prudent, and temperate. The utilities of the remaining life expectancies become $U(21) = 1.000$, $U(20) = 0.973$, $U(11) = 0.680$, and $U(2) = 0.244$ (note that $U(2)$ differs slightly from 0.25). The utility gain *G* and the utility loss *L* from treatment change to $G_A = 0.680 - 0.244 = 0.435$ and $L_A = 0.973 - 1.000 = -0.029$, leading to a critical prevalence rate of 6.23% ($\tilde{p}_A = -L_A/(G_A - L_A) = 0.029/(0.435 + 0.029)$). The risk-averse DM would therefore prescribe brain surgery at a much lower a priori probability. He values the utility gain to utility loss ratio $G/-L$ by a factor 1.67 higher than the risk neutral DM ($G_A/-L_A = 15$; $G_N/-L_N = 9$).

Next, we add a symmetric zero-mean comorbidity risk $E[\tilde{m}] = 0$, where $\tilde{m}$ can take on the values +1 or -1. Hence, the patient's life expectancy in the sick state, depending on the health status, becomes 3 or 1 with equal probability if untreated, and 12 or 10 if treated. For a risk neutral DM, nothing changes as the expected utility gain *G* remains at 9 ($G_N = 0.5 \cdot (12+10-3-1)$). For the prudent DM, the expected utility gain from treatment, $EG_P = (0.5/6.2) \cdot (12^{0.6} + 10^{0.6} - 3^{0.6} - 1^{0.6})$, increases to 0.443, which results in a higher utility

gain to utility loss ratio $EG_P/-L = 15.3$ and, thus, in a further decrease in the critical prevalence rate to $\tilde{p}_P = 6.13\%$.

Then, we consider a left-skewed comorbidity risk. Let $\tilde{m}$ take on the values -2 with a probability of 20% and 0.5 with a probability of 80%, so that the expected value and the variance of the background risk are the same as in the symmetric case. Again, nothing changes for the risk neutral DM. For the mixed risk-averse DM, however, the expected utility gain increases even further to $EG_T = 0.456$, leading to a higher utility gain to utility loss ratio $EG_T/-L = 15.8$ and a further reduction of the critical prevalence rate to $\tilde{p}_T = 5.97\%$.

Finally, we take into account that the physician can use a computer tomography (CT) scan with a sensitivity of 95% and a specificity of 97% for further diagnostics. Although the CT scan does the patient no harm, it produces false-positive and false-negative results which must be considered before using the test. The likelihood ratios for the CT scan amount to $LR^+ = 31.67$ and $LR^- = 0.05$. Using (11) for the test and treatment thresholds and the utility gain to utility loss ratios from treatment reported above, we obtain the following values: $\tilde{p}_N^{Dx} = 0.315\%$, $\tilde{p}_A^{Dx} = 0.209\%$, $\tilde{p}_P^{Dx} = 0.206\%$, $\tilde{p}_T^{Dx} = 0.200\%$, and $\tilde{p}_N^{Rx} = 65.99\%$, $\tilde{p}_A^{Rx} = 56.30\%$, $\tilde{p}_P^{Rx} = 55.89\%$, $\tilde{p}_T^{Rx} = 55.18\%$. We note that risk aversion in particular has a strong effect on the treatment threshold as compared to risk neutrality.

7 Discussion and Conclusion

In this paper we analyze the impact of higher-order risk preferences on decisions over medical testing and treatment. In particular, we investigate the effect of a left-skewed comorbidity risk in the presence of higher-order risk attitudes. We find that a risk-averse DM values the utility gain from treatment in the sick state more and the utility loss from treatment in the healthy state less than a risk neutral DM. When facing an exogenous comorbidity risk in the sick state, a risk-averse and prudent DM will value the utility gain even more. If the comorbidity risk is left-skewed, a temperate DM will value the utility gain from treatment in the sick state the highest.

Higher-order risk attitudes affect the test and treatment thresholds. Both thresholds decrease with an increasing utility gain from treatment, leading to earlier testing as well as earlier treatment. A risk-averse DM will thus test and treat earlier than a risk neutral DM. Facing a

background risk, a prudent and risk-averse DM will test and treat even earlier. And if the background risk is left-skewed, test and treatment thresholds decrease further yet for a risk-averse, prudent, and temperate DM.

Corresponding results apply for endogenous test outcomes. Since a risk-averse DM realizes a higher (lower) utility gain (loss) from treatment, he will lower the cutoff value to increase sensitivity at the expense of specificity. In the presence of background risk, prudent and temperate behavior (for left-skewed background risks) strengthens this effect, causing an even further decrease in the optimal cutoff.

Most of the commonly used utility functions (e.g. logarithmic and root functions) imply mixed risk aversion and have derivatives with progressively alternating signs (Brocket and Golden 1987; Caballé and Pomansky 1996). These utility functions model risk-averse, prudent, and temperate behavior at the same time. Recent experimental evidence confirms risk-averse and prudent behavior (Deck and Schlesinger 2010; Ebert and Wiesen 2011a and 2011b).[3] Empirical results for temperance are ambiguous so far. While Deck and Schlesinger (2010) find (weakly) intemperate behavior, the studies by Ebert and Wiesen (2011b) and Noussair et al. (2011) point in the other direction, in line with our theoretical results.

This paper has two limitations at least. First, our model is rooted in the EU framework, which has been criticized for its insufficient capacity to predict individual behavior, especially with regard to possible probability weighting. Alternatives to EUT, such as rank-dependent choice models, have been shown to be more in line with actual behavior (Abdellaoui 2000; Bleichrodt and Pinto 2000). Nonetheless, recent studies such as List (2004) challenge the validity of rank-dependent theory. They claim that experienced individuals behave largely in accordance with EUT. Since physicians routinely decide on medical tests and treatments, they should be able to make unbiased estimations of a priori probabilities of illnesses, and thus make coherent treatment decisions. Moreover, this study's nature is normative, and "expected utility is the best theory to determine which decisions to take" (Wakker 2008, p. 697).

The second limitation is the restriction to diagnostic risk. In general, decision makers face not only diagnostic risk but also therapeutic risk over the uncertain outcome of a treatment. Eeck-

[3] Deck and Schlesinger (2010) and Ebert and Wiesen (2011a) use the lotteries introduced by Eeckhoudt and Schlesinger (2006) and find that 61-65% of their subjects' responses are prudent. Ebert and Wiesen (2011b) also measure the intensity of prudence and find that "on average, the downside risk [prudence] compensation demanded is significantly higher than the second-order risk compensation" (p. 22).

houdt (2002) shows that risk aversion increases the critical success threshold in this case. The intuition here is that treatment is a risk increasing action, so that a risk-averse DM treats later than a risk neutral one. Therefore the effects of higher-order risk preferences on the thresholds may disappear when diagnostic and treatment risk are analyzed simultaneously.

Bleichrodt et al. (2003) analyze the effect of comorbidities on the treatment decision in the context of therapeutic risk. They mirror the QALY model and assume that treatment only affects quality of life, that comorbidity only impacts life duration, and that patients can influence treatment intensity. They find that comorbidities do not affect the treatment decision, since the prudence premium can be interpreted as the DM's risk premium for longer life. However, in contrast to Eeckhoudt's and our models, the background risk here is imposed on a second dimension (i.e. life duration) and not on the health level of the sick person.

Often pure health outcomes are used instead of utility measures, leading to excessively high test and treatment thresholds and cutoff values. The use of QALYs instead would reduce this bias, as QALYs reflect patient preferences over health statuses, thus reflecting higher-order risk preferences if present. However, often only the primary illness is taken into account when eliciting QALYs, but not the possibility of co-morbid conditions.

We conclude that medical decisions should be based on the accurate measurement of preferences. Our results are relevant for clinical guidelines. In particular, our findings suggest that – ceteris paribus – diagnostic tests in low prevalence settings, e.g. screenings for breast or prostate cancer, should be considered more beneficial if higher-order risk preferences are taken into account.

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References

Abdellaoui, M. (2000): Parameter-Free Elicitation of Utility and Probability Weighting Functions, *Management Science*, 46(11), 1497-1512.

Bleichrodt, H., Crainich, D. & Eeckhoudt, L. (2003): The effect of comorbidities on treatment decisions, *Journal of Health Economics*, 22, 805-820.

Bleichrodt, H. & Pinto, J. L. (2000): A Parameter-Free Elicitation of the Probability Weighting Function in Medical Decision Analysis, *Management Science*, 46(11), 1485-1496.

Brocket, P. L. & Golden, L. L. (1987): A Class of Utility Functions Containing All the Common Utility Functions, *Management Science*, 33(8), 955-964.

Caballé, J. & Pomansky, A. (1996): Mixed Risk Aversion, *Journal of Economic Theory*, 71, 485-513.

Deck, C. & Schlesinger, H. (2010): Exploring Higher Order Risk Effects, *Review of Economic Studies*, 77(4): 1403-1420.

Ebert, S. & Wiesen, D. (2011a): Testing for Prudence and Skewness Seeking, *Management Science*, 57(7): 1334–1349.

Ebert and Wiesen (2011b): *Joint Measurement of Risk Aversion, Prudence and Temperance*, Working paper, University of Bonn, Germany. http://www.uni-bonn.de/~eberts.

Eeckhoudt, L. (2002): *Risk and Medical Decision Making*, Kluwer – Boston.

Eeckhoudt L., Gollier, C. & Schlesinger, H. (1996): Changes in Background Risk and Risk Taking Behavior, *Econometrica*, 64(3): 683-689.

Eeckhoudt, L. & Schlesinger, H. (2006): Putting Risk in its Proper Place, *American Economic Review*, 96: 280-289.

Felder, S. & Mayrhofer, T. (2011): *Medical Decision Making – A Health Economic Primer*, Springer – Berlin.

Felder, S., Werblow, A. & Robra, B.-P. (2003): A-priori Risk and Optimal Test Accuracy in Prenatal Diagnostics, *Medical Decision Making*, 23(5): 406-413.

Fryback, D. G. & Lawrence, W. F. (1997): Dollars May Not Buy as Many QALYs as We Think: A Problem with Defining Quality-of-life Adjustments, *Medical Decision Making*, 17(3), 276-284.

Harris, R. A. & Nease Jr, R. F. (1997): The importance of patient preferences for comorbidities in cost-effectiveness analyzes, *Journal of Health Economics*, 16, 113-119.

Kimball, M. S. (1992): Precautionary Motives for Holding Assets, in: Newman, P., Milgate, M. & Falwell, J. (eds.), *New Palgrave Dictionary of Money and Finance*, MacMillan – London, 158-161.

Kimball, M. S. (1990): Precautionary Saving in the Small and in the Large, *Econometrica*, 58(1): 53-73.

List, J. (2004): Neoclassical Theory versus Prospect Theory: Evidence from the Market Place, *Econometrica*, 72(2), 615-625.

McNeil, B. J., Keeler, M. & Adelstein, S. M. (1975): Primer on Certain Elements of Medical Decision Making, *New England Journal of Medicine*, 293(5): 211-215.

Metz, C. E. (1978): Basic Principles of ROC Analysis, *Seminars in Nuclear Medicine*, 8(4): 283-298.

Noussair, C. N., Trautmann, S. T., G. van de Kuilen. 2011. Higher Order Risk Preferences, Demographics, and Financial Decisions. Working Paper, Tilburg University, Tilburg, The Netherlands. http://arno.uvt.nl/show.cgi?fid=114569.

Pauker, S. G. & Kassirer, J. P. (1980): The Threshold Approach to Clinical Decision Making. *New England Journal of Medicine*, 302(20): 1109-1117.

Pauker, S. G. & Kassirer, J. P. (1975): Therapeutic Decision Making: A Cost Benefit Analysis. *New England Journal of Medicine*, 293(5): 229-234.

Sox, H. S., Blatt, M. A., Higgins, M. C. & Marton, K. I. (2007): *Medical Decision Making*, Philadelphia – American College of Physicians. Butterworth-Heinemann – Boston. Reprint, originally published: 1988.

Wakker, P. P. (2008): Lessons Learned by (from?) an Economist Working in Medical Decision Making, *Medical Decision Making*, 28(5), 690-698.